Maximum Length Sequence

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It is often desirable to know the impulse response *h(t)* of linear systems such as rooms. All acoustical parameters can be derived from the impulse response between the source and the receiver position. With the *maximum length sequence method* (MLS method) it is possible to measure the impulse response with a great amount of accuracy and repeatability. The *maximum length sequence method* is based on the cross-correlation technique and thus highly immune to extraneous noise of all kinds. Even clicks, pops, footsteps etc. will all be transformed into benign noise distributed evenly over the entire impulse response. This property makes the method useful for acoustical measurements in very noisy environments.

The *maximum length sequence method* uses a maximum-length sequence—which is a periodic pseudo-random binary sequence—as the source signal. The binary sequence *x(k)* is represented by +1 and -1 and may be generated by a shift register with feedback.

From signal theory we know that the cross-correlation between the input *x(k)* and the output *y(k)* of a linear system, is related to the auto-correlation of the input by a convolution with the impulse response:

$$R_{xy}(k) = R_{xx}(k) * h(k)$$

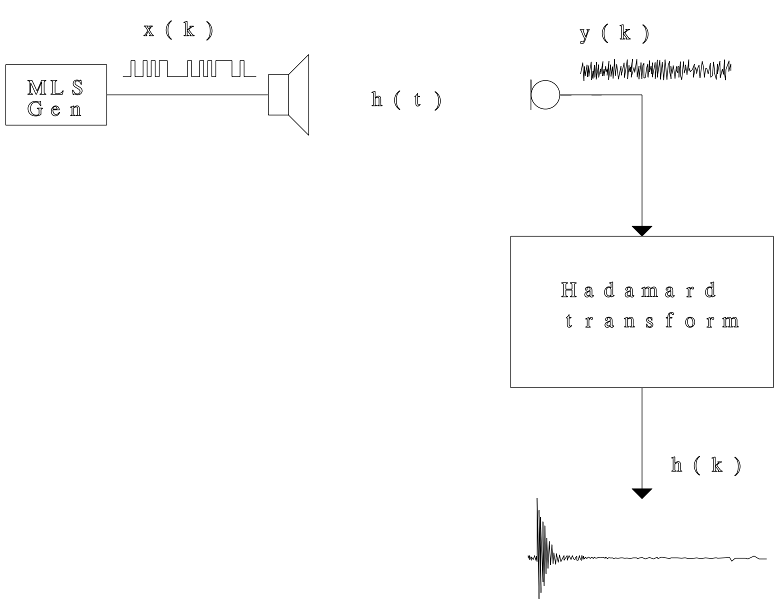
An important property of any MLS, is that its auto-correlation function is essentially an impulse. This impulse is represented by the *Dirac delta function:*

$$R_{xx}(k) \approx \delta(k)$$

The result of convolving a sequence with a *Dirac delta function*, is the sequence itself. Thus the impulse response *h(k)* can be found by cross-correlating the noise input *x(k)* with the output *y(k)*:

$$R_{xy}(k) \approx \delta(k) * h(k) = h(k)$$

Hence it is possible to measure the impulse response of linear systems by calculating the cross-correlation between the MLS and the system output signal. Since *x(k)* is a known pseudo-random sequence, there exists an efficient and very fast way to calculate the cross-correlation function $R_{xy}(k)$, called FAST HADAMARD TRANSFORM, also known as FHT.



A benefit of the FHT is that it requires, like the more familiar FAST FOURIER TRANSFORM, only $n \log_2(n)$ operations. Since the MLS is represented by +1 and –1, the FHT consists of additions and subtractions only.

An MLS is actually a deterministic signal, but it has similar spectral properties as true random white noise. A great advantage of the MLS technique is that since the sequence is deterministic, it can be repeated precisely. It is therefore possible to increase the signal-to–noise ratio (S/N) by synchronous averaging of the response sequences $y(k)$. Any extraneous, uncorrelated background noise will then be reduced for each averaging. The S/N ratio will increase by 3 dB for every doubling of the number of averages.

Since the MLS is a periodic signal, the autospectrum for the sequence consists of lines separated by the inverse of the period T_p. It can be shown that the lines for frequencies well below the clock frequency are equal in magnitude and that the spectrum thus approximates white bandlimited noise. Since the total power is independent of the clock frequency, the spectral density is increased by lowering the clock frequency.

If the sequence length is longer than half the reverberation time of a room, it can be shown that at least one spectral line will fall within every mode of the room. A common requirement is that the length of the MLS should be at least equal to the reverberation time.

Upper frequency f_h	Sampling frequency f_s	Sequence period T_p
$12.5\,\text{kHz} \leq f_h \leq 20\,\text{kHz}$	64 kHz	2.048 s
$6.3\,\text{kHz} \leq f_h \leq 10\,\text{kHz}$	32 kHz	4.096 s
$3.15\,\text{kHz} \leq f_h \leq 5\,\text{kHz}$	16 kHz	8.192 s
$1.6\,\text{kHz} \leq f_h \leq 2.5\,\text{kHz}$	8 kHz	16.384 s
$800\,\text{Hz} \leq f_h \leq 1.25\,\text{kHz}$	4 kHz	32.768 s
$0.1\,\text{Hz} \leq f_h \leq 630\,\text{Hz}$	2 kHz	65.536 s

Continuous Noise and Impulse Excitation

In the conventional method for the measurement of sound reduction and reverberation time, continuous noise is used for the excitation. *Schroeder* [9] has shown that the expected response $r(t)$ to a noise excitation switched off at the time $t=0$ is related to the impulse response h(t) by the following equation:

$$E[r^2(t)] = G\int_t^{\infty} h^2(u)du$$

in which G is a constant related to the excitation level.

A common application of this equation is to show that the reverberation decay may be computed from the impulse response. Another implication is that the expected level $E[r^2(0)]$ due to the continuous noise excitation is obtained by integration of the impulse response over the observed time.

The impulse response may therefore be used to obtain the reverberation decay as well as the stationary level from the noise excitation.

Implementation

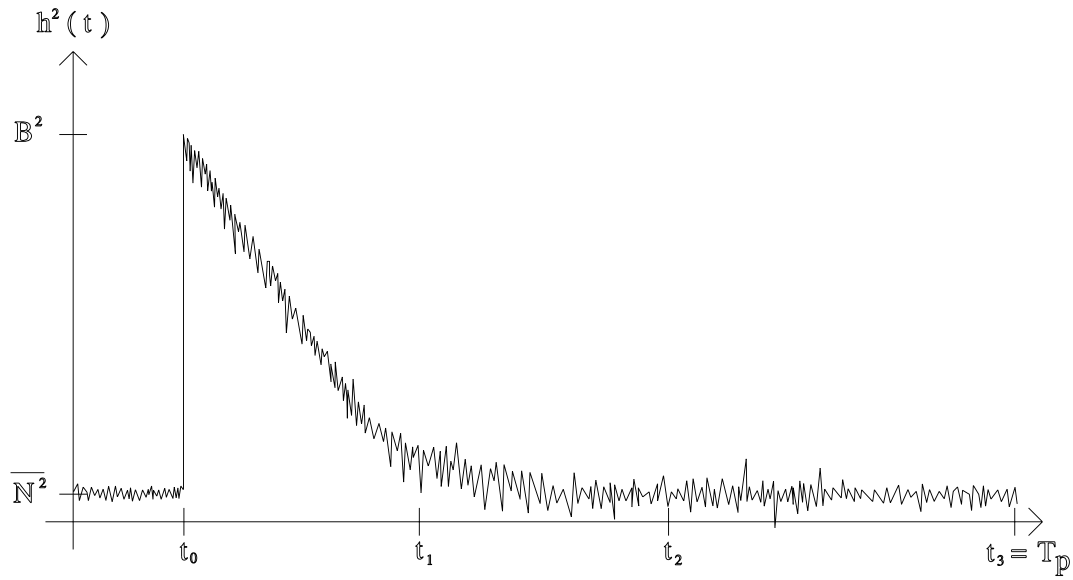
The MLS method is implemented in the Nor-840 with some restrictions to make it easy to use even if the user doesn't really know the details of the technique.

The length of the MLS is fixed to $2^{17}-1$, which gives an impulse response of 131071 samples. It is possible to change the duration T_p of the MLS by changing the sampling frequency f_s. When changing f_s, we have to satisfy the *Nyquist* sampling theorem. Therefore we have chosen to let the upper frequency f_h select the sampling frequency. The relationship between f_h and f_s is shown in the below table.

When making an MLS measurement with the RTA840, the following will be achieved:

The MLS is used as the excitation signal and is available at the generator output socket. To eliminate initial transients, the measurement starts after one MLS period, or in other words, after T_p seconds.

If the *Number of averages* is greater than 1, the responses *y(k)* of the MLS will be synchronously averaged in order to reduce the influence of the background noise.



When the measurement is finished, the post-processing tasks will be started. The response *y(k)* will first be transformed by the HADAMARD TRANSFORM to obtain the broadband impulse response *h(k)*. This impulse response is used as the input signal for a normal multi-spectrum measurement in the RTA840. When this multi-spectrum measurement has been made, the signal processing of the results will start. The signal processing part can be divided into two groups; one for level measurements and one for reverberation time measurements.

Level Measurements

Level measurements are carried out in single-spectrum mode. The level for each octave or one-third octave band can be calculated by integrating the multi-mode levels from $t = 0$ to $t = T_p$. By doing so, we will integrate all the background noise as well, and we will not achieve the full enhancement of the S/N ratio compared to the conventional measurement method. The HADAMARD TRANSFORM will collect almost all of the signal energy in the first part of the impulse response. If we only integrate the part of the impulse response where the signal energy is significant, and skip the part where the background noise dominates, we will get a better estimate of the signal. If we integrate the last part of the multi-spectrum measurement, we will get an estimate of the background noise level.

The signal level $S_1{}^2$ and the noise level N^2 can thus be determined by:

$$S_1^2 = \int_0^{t_1} h^2(t)dt$$

$$\overline{N^2} = \frac{1}{t_3 - t_2}\int_{t_2}^{t_3} h^2(t)dt$$

The background noise can also be subtracted from the signal to estimate the level of the signal *without* background noise. The signal level S^2 will be given by:

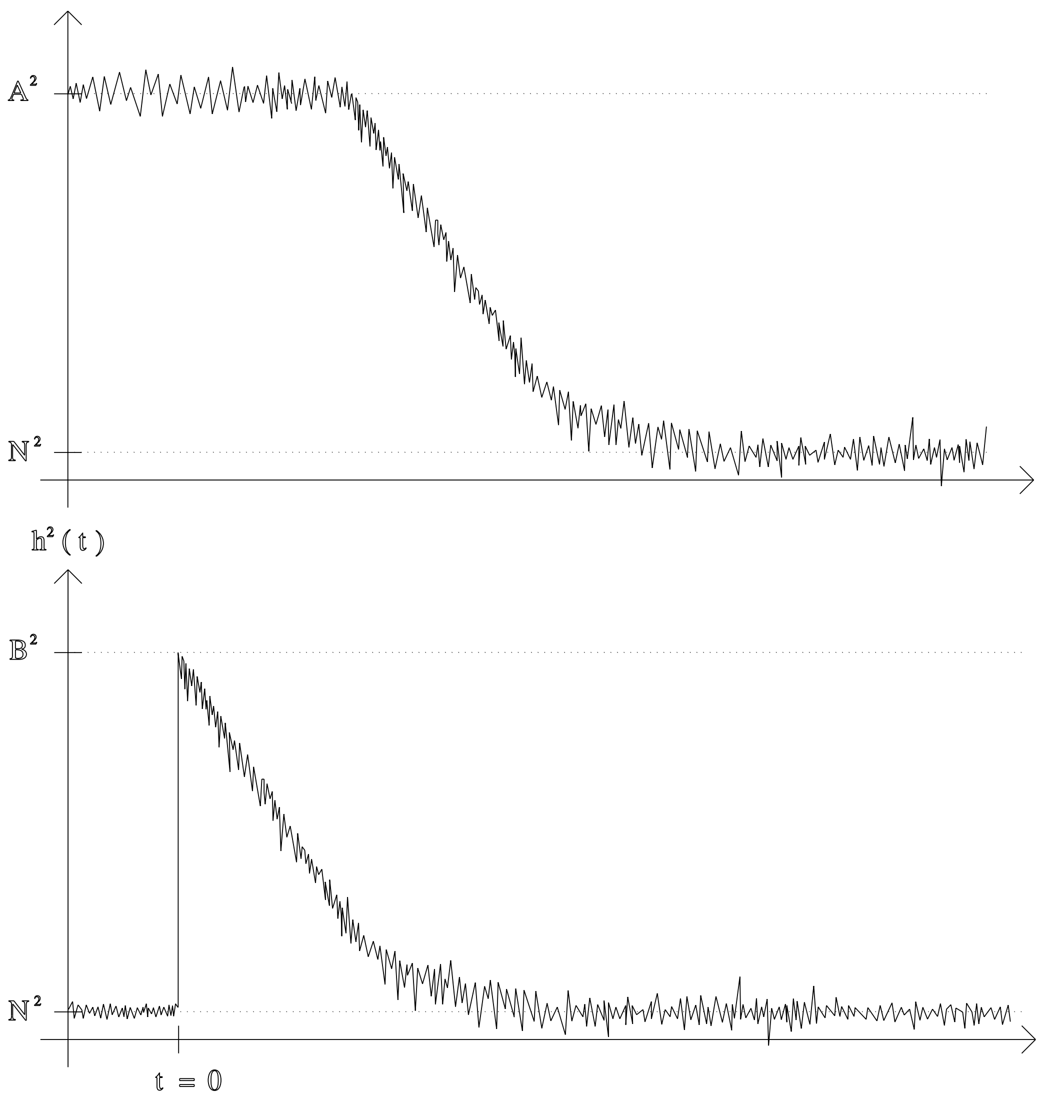
$$S^2 = \int_0^{t_1} (h^2(t) - \overline{N^2})dt = S_1 - t_1\overline{N^2}$$

The integration limit t_1 is chosen to be the intersection point between the signal decay and the background noise.

Since almost all of the signal energy is present in the early part of the decay, it is not necessary to integrate more than
$T_{Rev}/2$ of the decay (which is the point where the level of the impulse response has decreased by 30 dB). To ensure that we integrate a sufficient part of the signal energy, we say that we have to integrate at least $T_{Rev}/3$ of the decay. The integration limit t_1 is then given by:

$$\frac{T_{Rev}}{3} \leq t_1 - t_0 \leq \frac{T_{Rev}}{2}$$

The integration limits will be selected individually for each one-third octave or octave band to achieve the optimum result for each band dependent on the background noise present.



Since the background noise is automatically subtracted from the level measurement, *one should refrain from any further corrections for the background noise.*

S/N-ratio Enhancement of Level Measurements

It is possible to estimate the S/N-ratio enhancement, related to conventional measurement methods, if we make use of the property that the energy of a signal will not be changed by the HADAMARD TRANSFORM.

The conventional S/N-ratio will be:

$$\left(\frac{S}{N}\right)^2 = \frac{\int_0^{T_p} s^2(t)dt}{\int_0^{T_p} n^2(t)dt}$$

in which $s(t)$ is the signal before the HADAMARD transformation has been applied, and $n(t)$ is the noise before the HADAMARD transformation has been applied.

If we set the integration length
$T_H = t_1$, we can assume that all energy of s_H will be present within the period T_H:

$$\int_0^{T_p} s^2(t)dt \approx \int_0^{T_H} s_H^2(t)dt$$

in which $s_H(t)$ is the signal *after* the HADAMARD transformation.

The energy of the extraneous noise will—after the HADAMARD transformation—still be distributed evenly over the entire period T_p.

$$\int_0^{T_H} n_H^2(t)dt \approx \frac{T_H}{T_p}\int_0^{T_p} n^2(t)dt$$

$$\left(\frac{S}{N}\right)_H^2 \approx \left(\frac{S}{N}\right)^2 \cdot \frac{T_p}{T_H}$$

In which $n_H(t)$ is the noise *after* the HADAMARD transformation.

The S/N-ratio of an MLS measurement is then given by:

$$\left(\frac{S}{N}\right)_H \approx \left(\frac{S}{N}\right) \cdot \sqrt{\frac{T_p}{T_H}}$$

The S/N-ratio will increase even more if we select *Number of averages* (M) to be greater than 1. The total measurement time will then be $T_{tot} = M \cdot T_p$ and the S/N-ratio will be:

$$\left(\frac{S}{N}\right)_H \approx \left(\frac{S}{N}\right) \cdot \sqrt{\frac{M \cdot T_p}{T_H}} = \frac{S}{N} \cdot \sqrt{\frac{T_{tot}}{T_H}}$$

The enhancement in dB will be given by:

$$10\log\left(\frac{T_{tot}}{T_H}\right)$$

or, as T_H is less than half the reverberation time T_{Rev}:

$$\Delta_{S/N} > 10\log\left(\frac{T_{tot}}{T_{Rev}}\right) + 3dB$$

Reverberation Time Calculations

The reverberation time calculations are based on short time L_{eq} values obtained by filtering the broadband impulse response h(t). The reverberation time parameters EDT, T_{20} and T_{30}, are calculated from the filtered impulse response by application of the SCHROEDER method. The backward integration starts at the intersection point between the linear signal decay and the background noise level. This starting point will be selected individually for each frequency band to achieve optimal dynamic range.

- T_{20} is measured as time between the –5 dB and –25 dB crossing
- T_{30} is measured as time between the –5 dB and –35 dB crossing
- EDT is measured as time between the –1 dB and –11 dB crossing

S/N-ratio Enhancement of Reverberation Time Calculations

The S/N-ratio of a conventional reverberation time calculation, based on interrupted noise, is given by:

$$\left(\frac{S}{N}\right)^2 = \frac{A^2}{N^2}$$

in which A^2 and N^2 is the square of the response with and without excitation respectively, as shown in the top-most Fig. below.

For an MLS derived impulse response, the expected level at time t will be (from *Schroeder* [9]):

$$E\{r^2(t)\} = \int_t^{\infty} h^2(t)dt$$

For $t > 0$, the impulse response $h(t)$ may be approximated by an exponential decay.

$$h^2(t) \approx B^2 e^{-\frac{t}{\tau}}$$

The level of the backward integrated curve, at $t = 0$, will then be:

$$E\{r^2(0)\} \approx \int_0^{\infty} B^2 e^{-\frac{t}{\tau}} dt = B^2\tau$$

If the MLS is used as a conventional noise excitation signal, the response will have the mean value A^2. The total energy in one sequence period will therefore be $A^2 \cdot T_p$. If the response is HADAMARD transformed to obtain the impulse response, the same energy will be present in this response. The relation between the values A^2 and B^2 is therefore:

$$A^2 T_p = \int_0^{\infty} h^2(t)dt \approx B^2\tau$$

The level of the noise floor after the HADAMARD transform will be as it was before (extraneous, uncorrelated noise).

The dynamic range for the reverberation calculation, based on the MLS method, may be defined as the ratio between the maximum level of the impulse response and the noise floor:

$$\left(\frac{S}{N}\right)_H^2 = \frac{B^2}{N^2}$$

The S/N-ratio enhancement compared to the conventional interrupted noise method, will then be:

$$\frac{\frac{B^2}{N^2}}{\frac{A^2}{N^2}} = \frac{B^2}{A^2} = \frac{T_p}{\tau}$$

The time-constant τ can be expressed as a function of T_{Rev}:

$$\tau = \frac{T_{Rev}}{\ln(10^6)} = \frac{T_{Rev}}{13{,}8}$$

This gives a S/N-ratio enhancement in dB:

$$10\log\left(\frac{T_p}{\tau}\right) = 10\log\left(\frac{T_p}{T_{Rev}}\right) + 11{,}4dB$$

If the *Number of averages* (M) is selected to be greater than 1, the
S/N-ratio will be related to the total measurement time $T_{tot} = M \cdot T_p$. S/N-ratio enhancement in dB will then be:

$$10\log\left(\frac{T_{tot}}{T_{Rev}}\right) + 11{,}4dB$$

Limitations

Finally, we are going to look into the limitations of the MLS technique.

Crosstalk

The MLS technique allows measurements with large dynamic ranges. Even inherent noise in microphones and measurement systems are reduced below the normal values. Care must therefore be taken to eliminate influence from unwanted signal paths such as electrical crosstalk. Cables for the excitation, such as loudspeaker cables, should be located far away and screened from the microphone cables. Even internal crosstalk in the instrumentation, normally buried in the self-noise, may show up.

Crosstalk may be investigated by replacing the microphone with a dummy microphone. The measurement values should then decrease significantly. A signal caused by electrical crosstalk will have very short reverberation time and may be found by studying the impulse response.

Time-invariance

The MLS may be described as a series of pulses of equal amplitude but appearing in positive or negative direction in a random manner. The measured signal will be the response to these pulses. The effect of the HADAMARD transformation is to transfer the individual responses at different times and in different direction so they appear as a response to impulses appearing simultaneous and in one direction. It is therefore important that the system remains unchanged during the measurement. The requirement to time-invariance is very important and should always be considered when MLS techniques are applied.

In building acoustics it is common to move the microphone during a level measurement to obtain spatial averaging. *This method cannot be used when correlation or MLS techniques are applied*. If you do so, responses with different phases will be averaged together and the mean value will be too low. This will happen if more periods of the sequence is averaged, but the effect will also apply for a measurement based on one period of the sequence only.

Elecroacoustic components such as microphones, loudspeakers, power amplifiers and measurement channels are normally so stable that few problems due to time-invariance are to be expected. The acoustical transmission may however cause problems. The transmission of sound is sensitive to changes in the environmental conditions such as temperature, humidity and windspeed. The temperature and humidity in a room are normally changing slowly. Time-invariance due to temperature and humidity is therefore more pronounced if the measurement is performed over a long period of time. Few problems related to change in atmospheric pressure, temperature and humidity are reported in the literature, but the phenomenon should always be considered when the MLS method is applied.

Time-invariance due to wind shows normally short-time fluctuation and is always a problem for the MLS technique. As the errors are caused by averaging impulse responses where the phase is not stable, the effect is most pronounced at higher frequencies. The wind will normally influence the sound transmission by altering the speed of the sound. The phase-error will therefore increase with the sound path length. In a room the last part of the reverberation decay will be more sensitive to this effect and the recorded decay will decrease too fast in the last part of the decay.

Outdoor measurements with MLS should be made with outmost care. Normally, the method cannot be applied to measure sound transmission over large distances.

Linearity

The theory requires that the system has to be linear in order to be valid. The effect of unlinearities has been discussed in a number of papers (ref [6], [7] and [14]). The analysis shows that if the system includes unlinear terms, the impulse response will not decrease asymptotically to zero as it should for a linear system. The effect will therefore be seen in the impulse response as unregular noise. However, unlike noise this error will not decrease by further averaging.

Analysis, as well as experiments, reveals that the MLS method is not very sensitive to unlinearities and that the sensitivity will decrease by applying longer sequences. For room and building acoustics measurements, the sequence is normally long. Minor unlinearities in the magnitude of 1–3 % will therefore not influence a level measurement if the integration period is limited to about half the reverberation time. The unlinearity may limit the dynamic range for reverberation curves. However, as the errors show up as noise in the reverberation curve, normal detection of background level errors will indicate this type of problems.

Conclusions

It is shown that measurement of airborne sound reduction and reverberation time related quantities may be measured by application of the MLS-technique with advantage. Even if not mentioned in the standards, the method should be considered as a practical way to obtain a similar result as obtained by using the standardized noise excitation for level difference measurements and abrupt noise excitation for the reverberation time. The implementation in the RTA840 instrument ensures that instrument standards related to linearity, filtering and signal detection are fulfilled. We therefore recommend that the MLS method should be allowed as a viable extension within the present formulation of the standards provided that it is implemented in a compatible way.

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